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A simulation on the public good provision under various taxation systems

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Abstract

This paper integrates two traditions in taxing theory by constructing a primitive general equilibrium (GE) model which incorporates a public good, and examine the desirability of taxing system to sustain its optimum level. Formally, we start with utilizing the Lindahl mechanism to compute a Pareto-optimal public good level under a specification of the parameters on production and utility function, with k the substitution parameter on the latter. The burden-sharing in this Lindahl mechanism is called the Lindahl tax. We compute the rates of various taxes in order to sustain the optimal public good level, and compare the Gini coefficients and the social welfares. It is shown for a specified case that when $0<k<1$, there exists no general equilibrium for the poll tax case, while the income tax (and proportional commodity tax) is more desirable than the Lindahl tax from the viewpoint of Gini coefficient and also from the utilitarian social welfare viewpoint. Next, selecting parameters on production and utility functions and initial endowments randomly, we show that the property is robust, provided that $0<k<1$. However, when $k<0$, the same simulation shows that the Lindahl tax is more desirable than the income tax (and proportional commodity tax) from the two viewpoints. Finally, it is shown that when $0<k<1$, the Walrasian tatonnement process to compute the income tax is globally stable, while the one when $k<0$ is locally unstable. Thus, this paper concludes that the income tax (and the proportional commodity tax) tends to be superior to the Lindahl tax in order to provide public good.

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Keywords: *general equilibrium*; *public good*; *Lindahl mechanism*; *income tax*; *commodity tax*; *poll tax*; *Walrasian tatonnement*

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1. Introduction

The aim of this paper is to extend Fukiharu1, whose aim was to integrate two traditions of taxation problem. On the one hand, the argument on taxation is constructed in such a way that which taxation is desirable in order to impose a tax, income tax, commodity tax or poll tax, without paying any attention on why the tax is necessary. In the partial equilibrium framework, Stiglitz2, Chapter 17, regards the lump-sum tax, such as poll tax, the best. On the other hand, it is also a tradition to assert that government imposes a tax in order to provide public goods (Stiglitz2, Chapter 4). Fukiharu1 combined the above two traditions by constructing a primitive general equilibrium (GE) model which incorporates a public good, asking which taxation is desirable in order to impose tax, income tax, commodity tax, or poll tax to provide the public good. In this paper, we first summarize Fukiharu1 in two parts. In Part I, we utilize the Lindahl mechanism to compute a Pareto-optimal public good level under a specification of parameters. The burden-sharing in this Lindahl mechanism may be regarded as a tax on the society members, while utilizing pseudo-market mechanism. We call it Lindahl tax. Next, we compute the rate of income tax in order to sustain the optimal public good level, and compare the Lindahl tax and income tax. We proceed to the computation of the rate of (proportional) commodity tax in order to sustain the optimal public good level, and compare the Lindahl tax, income tax and commodity tax. Finally we proceed to the comparison between the Lindahl tax, income tax, commodity tax and poll tax under the specification of the model. In Part II, selecting the parameters randomly, we examine the robustness of the conclusion in Part I. The conclusion in Fukiharu1 was not strong, in that the conclusion crucially depends on the substitution parameter of the utility function. Thus, in this paper, we add Part III, in which the Walrasian process to compute the various taxes is constructed in order to examine whether stability is guaranteed. The Part III follows the tradition of comparative statics in economics: i.e. conclusions must be compatible with market stability (Samuelson3).

2. Economy with public good and Walras-Lindahl mechanism: specified GE model: part I (1)

We start with constructing a primitive GE model which incorporates a public good. The production and utility functions are specified by particular functions, and their parameters are also specified. Utilizing Walras-Lindahl mechanism, the optimal level of public good, as well as burden-sharing of each member of the society, is derived.

2.1. Assumptions on production side

Country A is under national isolation. She has three sectors of production, which produces 3 goods, utilizing labor, L_i, and capital, K_i: y stands for the output of sector 1, x stands for the output of sector 2, and z stands for the output of sector 3 (i=1, 2, 3), where y and x are private goods and z is a public good. Production function of sector 1, $y=f_1=L_1^{a1}K_1^{b1}$, with $a_1+b_1<1$: *decreasing returns to scale*, is specified by $f_1=L_1^{1/6}K_1^{1/5}$. Production function of sector 2, $x=f_2=L_2^{a2}K_2^{b2}$, with $a_2+b_2<1$: *decreasing returns to scale*, is specified by $f_2=L_2^{1/4}K_2^{1/3}$. Production function of sector 3, $z=f_3=L_3^{a3}K_3^{b3}$, with $a_3+b_3=1$: *constant returns to scale*, is specified by $f_3=L_3^{1/3}K_3^{2/3}$. From the profit maximization of the sector 1, demand for labor, L_1^{D}, and demand for capital, K_1^{D}, are computed with p_y, the price of the consumption good, y, w_L, the wage rate of labor, and w_K, the rental price of capital, as parameters. Thus, the supply function of y, y^S, is computed with p_y, w_L, and w_K, as parameters. The profit function of sector 1, π_1, is computed, with p_y, w_L, and w_K, as parameters. This profit accrues to entrepreneur 1. From the profit maximization of the sector 2, demand for labor, L_2^{D}, and demand for capital, K_2^{D}, are computed with p_x, the price of the consumption good, x, w_L and w_K as parameters. Thus, the supply function of x, x^S, is computed with p_x, w_L, and w_K, as parameters. The profit function of sector 2, π_2, is computed, with p_x, w_L, and w_K, as parameters. This profit accrues to entrepreneur 2. The sector 3 produces a public good, z, under *constant returns to scale*, so that demand for capital, K_3^{D}, and the one for labor, L_3^{D}, is derived by the minimizing cost, given output level z, with w_L and w_K, as parameters. The price of the public good, p_z, is determined so that $p_z z= w_L L_3^{\mathrm{D}}+w_K K_3^{\mathrm{D}}$.

2.2. Assumptions on consumption side

We proceed to the demand side of country A. She is endowed with the initial labor, L_e, and the initial capital, K_e. In this paper, the aggregate worker possesses α_L of L_e and β_L of K_e, while the aggregate capitalist possesses α_K of L_e and β_K of K_e, where $\alpha_L+\alpha_K=1$ and $\beta_L+\beta_K=1$. It is specified in this section that $L_e=100$, $K_e=50$, $\alpha_L=1$, $\alpha_K=0$, $\beta_L=0$, and $\beta_K=1$. All the agents in this paper: (aggregate) workers, (aggregate) capitalists, and 2 entrepreneurs, have the same CES utility function, $u[y, x, z] = (\gamma_y y^k+\gamma_x x^k+\gamma_z z^k)^{1/k}$ which is specified as $u[y, x, z] = (y^{1/2}+x^{1/2}+z^{1/2})^2$: *i.e.* $k=1/2$, $\gamma_y=\gamma_x=\gamma_z=1$. All the consumers maximize utility subject to income constraint:

$$\max u[y, x, z] \text{ s.t. } p_y y+p_x x+\theta_j p_z z=m_j \ (j=L, K, 1, 2) \tag{1}$$

where m_j is income and θ_j is the burden share of the household j for the public good ($j=L, K, 1, 2, 3$). The aggregate workers (household L)'s income, m_L, consists of initial endowment of labor, evaluated by the wage rate: $w_L L_e$. It is assumed that they supply L_e for labor supply. The aggregate capitalists (household K)'s income, m_K, consists of initial endowment of capital, evaluated by the rental price of capital: $w_K K_e$. It is assumed that they supply K_e for capital supply. Entrepreneur 1 (household 1)'s income, m_1, consists of the profit for sector 1, π_1. Finally, entrepreneur 2 (household 2)'s income, m_2, consists of the profit for sector 2, π_2.
From (1) the demand function of workers for commodity y, y_L^D, that for commodity x, x_L^D, that for commodity z, z_L^D, the demand function of capitalists for commodity y, y_K^D, that for commodity x, x_K^D, that for commodity z, z_K^D, the demand function of entrepreneur 1 for commodity y, y_{E1}^D, that for commodity x, x_{E1}^D, that for commodity z, z_{E1}^D, and finally the demand function of entrepreneur 2 for commodity y, y_{E2}^D, that for commodity x, x_{E2}^D, that for commodity z, z_{E2}^D are derived.

2.3. General equilibrium with public good: Walras-Lindahl equilibrium with Lindahl tax

General equilibrium for country A with public good, "GE with public good", or Walras-Lindahl equilibrium, is defined by the following system of simultaneous equations.

$$y_L^D + y_K^D + y_{E1}^D + y_{E2}^D = y^S \tag{2}$$
$$x_L^D + x_K^D + x_{E1}^D + x_{E2}^D = x^S \tag{3}$$
$$z_L^D = z_K^D = z_{E1}^D = z_{E2}^D = z \tag{4}$$
$$L_1^D + L_2^D + L_3^D = L_e \tag{5}$$
$$K_1^D + K_2^D + K_3^D = K_e \tag{6}$$

From the application of Newton method on (2), (3), (4) and (6) we compute the GE with public good, which satisfies (5). This "GE with public good" with the optimum level $z^O= 56.385556$, is also derived by the following Walras-Lindahl differential equations, in which t stands for time.

$$\begin{aligned}
&\mathrm{d}p_y[t]/\mathrm{d}t= y_L^D + y_K^D + y_{E1}^D +y_{E2}^D - y^S\\
&\mathrm{d}p_x[t]/\mathrm{d}t= x_L^D + x_K^D + x_{E1}^D + x_{E2}^D - x^S\\
&\mathrm{d}w_K[t]/\mathrm{d}t= K_1^D + K_2^D + K_3^D - K_e\\
&\mathrm{d}\theta_L[t]/\mathrm{d}t= z_L^D - (z_L^D + z_K^D + z_{E1}^D + z_{E2}^D)/4\\
&\mathrm{d}\theta_K[t]/\mathrm{d}t= z_K^D - (z_L^D + z_K^D + z_{E1}^D + z_{E2}^D)/4\\
&\mathrm{d}\theta_1[t]/\mathrm{d}t= z_{E1}^D - (z_L^D + z_K^D + z_{E1}^D + z_{E2}^D)/4\\
&\mathrm{d}z[t]/\mathrm{d}t= z_{E2}^D - z[t]
\end{aligned} \tag{7}$$

The set of GE incomes after the deduction of *Lindahl tax* (payment for the consumption of public good), $\{m_L^*, m_K^*, m_{E1}^*, m_{E2}^*\}$, and the one of GE utility levels, $\{u_L^*, u_K^*, u_{E1}^*, u_{E2}^*\}$ are computed easily.
The Gini coefficients before and after the Lindahl tax, $Gini^{L0}$ and $Gini^L$, are given respectively as in what follows.

$$Gini^{L0}=0.473504,\ Gini^L =0.605159 \tag{8}$$

2.4. General equilibrium with public good: income tax provision of public good

In this subsection, we examine if it is possible to use income tax instead to Lindahl tax in order to achieve the optimum public good level, z^O= 56.385556. Since z^O is provided for each member of the society, the utility function becomes the following one $u[y, x, z^O]$.

All the consumers maximize utility subject to income constraint:

$$\max u[y, x, z^O] \text{ s.t. } p_y y+p_x x=(1-\tau_I)m_j \quad (j=L, K, 1, 2) \tag{9}$$

One of the GE conditions, (4), is replaced by the following.

$$p_z z=\tau_I\,(w_L\,L_e+ w_K\,K_e+ \pi_1+ \pi_2\,) \tag{10}$$

Utilizing the Newton method, we obtain the GE prices and tax rate τ_I= 0.807053.

It is ascertained that we have exactly the same GE prices as in the Lindahl tax case. The set of GE incomes after the deduction of *income tax*, $\{m_L{*}^I, m_K{*}^I, m_{E1}{*}^I, m_{E2}{*}^I\}$, and the one of GE utility levels, $\{u_L{*}^I, u_K{*}^I, u_{E1}{*}^I, u_{E2}{*}^I\}$ are computed easily

From the viewpoint of *Bentham-type utilitarian*, the income tax is more desirable than the Lindahl tax, since the sum of utility for the former case is greater than the one for the latter case, as shown in what follows.

$$u_L{*}^I+u_K{*}^I+u_{E1}{*}^I+u_{E2}{*}^I= 306.468869> 296.859740= u_L{*}+u_K{*}+u_{E1}{*}+u_{E2}{*}$$

Furthermore, the income tax is more desirable than the Lindahl tax in the sense that the Gini coefficient for the former, $Gini^I$ is smaller than the latter case, as shown in what follows.

$$Gini^I =0.473504 \tag{11}$$

It is also ascertained that the Gini coefficient for income tax is exactly the same as the pre-tax Gini coefficient for Lindahl tax case.

2.5. General equilibrium with public good: proportional commodity tax provision of public good

In this subsection, we examine if it is possible to use the *proportional* commodity tax instead to Lindahl tax in order to achieve the optimum public good level, z^O.

All the consumers maximize utility subject to income constraint:

$$\max u[y, x, z^O] \text{ s.t. } (1-\tau_C)p_y y+(1-\tau_C)p_x x=m_j \quad (j=L, K, 1, 2) \tag{12}$$

One of the GE conditions, (4), is replaced by the following.

$$p_z z=\tau_C\,p_y\,(y_L{}^{\mathrm{D}}+y_K{}^{\mathrm{D}}+y_{E1}{}^{\mathrm{D}}+y_{E2}{}^{\mathrm{D}}) + \tau_C\,p_x\,(x_L{}^{\mathrm{D}}+x_K{}^{\mathrm{D}}+x_{E1}{}^{\mathrm{D}}+x_{E2}{}^{\mathrm{D}}) \tag{13}$$

It is confirmed that exactly the same GE prices utilities and incomes are derived as in the income tax case except for the proportional commodity tax rate, τ_C= 4.18278. Thus, the *proportional* commodity tax is more desirable than the Lindahl tax. Note that the equality of the income taxation and the *proportional* commodity taxation on GE was noticed in Shoven and Whalley[4].

2.6. General equilibrium with public good: poll tax provision of public good

In this subsection, we examine if it is possible to use poll tax instead to Lindahl tax in order to achieve the optimum public good level, z^O.

All the consumers maximize utility subject to income constraint:

$$\max u[y, x, z^O] \text{ s.t. } p_y y + p_x x = (m_j - T/4) \quad (j=L, K, 1, 2) \tag{14}$$

where m_j is pre-tax income and T is the tax to sustain z^O ($j=L, K, 1, 2$) . One of the GE conditions, (4), is replaced by the following.

$$p_z z = T \tag{15}$$

It is confirmed that exactly the same GE prices and utilities are derived as in the income tax case except for the poll tax, T= 259.964421. It must be noted, however, that the poll tax cannot sustain z^O, since the income after the poll tax is negative for entrepreneurs 1 and 2.

$\pi_1 - T/4 = -48.327516$, $\pi_2 - T/4 = -50.057708$

(The computation in Section 2 was conducted in Fukiharu[5]).

3. Robustness of the specified GE model: part I (2)

In this section, we examine the robustness of the conclusion in Section 2. First, modification of parameters is made, in order to examine the conclusion. Suppose that a_1=2/3, b_1 =1/8, a_2=1/2, b_2 =1/3, a_2=3/5, b_3 =2/5, L_e=100, K_e=50, α_L=2/3, α_K=1/5, β_L=1/3, and β_K=4/5, k=1/2, $\gamma_y=\gamma_x$=1, and γ_z=1/100.

3.1. General equilibrium with public good: Walras-Lindahl equilibrium with Lindahl tax

The Gini coefficients before and after the Lindahl tax and the Bentham-type utilitarian social utility level are given respectively as in what follows.

$Gini^{L0}$=0.3309739379674684046, $Gini^{L}$ = 0.3310327896068688795
$u_L^* + u_K^* + u_{E1}^* + u_{E2}^*$=84.53727799264331946

3.2. General equilibrium with public good: income tax provision of public good

The Gini coefficients and the Bentham-type utilitarian social utility level are given respectively as in what follows.

$Gini^I$ = 0.3309739379674684046= $Gini^{L0} < Gini^L$
$u_L^{*I} + u_K^{*I} + u_{E1}^{*I} + u_{E2}^{*I}$ = 84.53727972251664468> $u_L^* + u_K^* + u_{E1}^* + u_{E2}^*$

Thus, we have the same conclusion as in Section 2: the income tax is more desirable than the Lindahl tax.

3.3. General equilibrium with public good: proportional commodity tax provision of public good

We have exactly the same conclusion as in section 2: the proportional commodity tax produces exactly the same GE prices and quantities.

3.4. General equilibrium with public good: poll tax provision of public good

In comparison with section 2, the optimum public good level, z^O=0.038835 is so small, and the poll tax, T= 0.069481 is also so small, that z^O can be supported by T. Every member has positive income after paying the poll tax. The Gini coefficient, $Gini^P$, and the Bentham-type utilitarian social utility level, $u_L^{*P} + u_K^{*P} + u_{E1}^{*P} + u_{E2}^{*P}$, are given respectively as in what follows.

$Gini^P = 0.3311077313088771057 > Gini^L > Gini^I$
$u_L*^P+u_K*^P+u_{E1}*^P+u_{E2}*^P = 84.53727576387538624 < u_L*+u_K*+u_{E1}*+u_{E2}* < u_L*^I+u_K*^I+u_{E1}*^I+u_{E2}*^I$

Thus, the income tax is the best taxation and the poll tax is the worst taxation. (The computation in Section 3 was conducted in Fukiharu[5].)

4. Simulations: part II

The analysis in Section 2 and 3 might suggest that the income tax and the proportional commodity tax are the most desirable taxation. It must be noted, however, that k=1/2 is assumed. If k is selected differently it is not clear that the above conclusion is robust. Therefore simulations are necessary with k selected differently from 1/2.

4.1. Simulation when 0<k<1

In this subsection, we conduct a simulation to compare the desirability of taxes to provide the public good by selecting parameters randomly when $0<k<1$.

In what follows, first, 100 tuples of parameters for $\{a_1, b_1, a_2, b_2, a_3, b_3, L_e, K_e, \alpha_L, \alpha_K, \beta_L, \beta_K, k, \gamma_y, \gamma_x, \gamma_z\}$ are selected randomly, where $a_i+b_i<1$, i =1, 2, $a_3+b_3=1$, $\alpha_L+\alpha_K=1$, $\beta_L+\beta_K=1$, and $0<k<1$ are satisfied and a_i, b_i and α_L etc. and k are expressed by n/m for integers n and m which belongs to [1, 10], L_e and K_e are integers belonging to [1, 1000], and γ_y, γ_x, γ_z are integers belonging to [1, 10]. Next, we apply them to the *Mathematica* program to compute GE prices, tax rates, utilities, and incomes. Among 100 simulations only 65 simulations satisfy required 22 equilibrium conditions. The reason for the smallness of the number stems from the Newton method itself, stability of the process on which crucially depends on the initial position. In order to raise the number, we must search for the initial position which guarantees the stability of the process for each simulation. Unfortunately, however, in this paper fixed initial position is utilized. Among 65 "successful" simulations, 59 cases satisfied $Gini^L > Gini^I$ and $u_L*+u_K*+u_{E1}*+u_{E2}* < u_L*^I+u_K*^I+u_{E1}*^I+u_{E2}*^I$. In other words, 90% of the "successful" simulations guaranteed the conclusion in Section 2. Among the "unsuccessful" simulations, there existed some complex-number-income solutions for the poll tax, so that the comparison between the poll taxation and other taxations was not done.

We repeated this simulations 50 times. The following data shows the shares of the cases which satisfied $Gini^L > Gini^I$ and $u_L*+u_K*+u_{E1}*+u_{E2}* < u_L*^I+u_K*^I+u_{E1}*^I+u_{E2}*^I$ among the "successful" simulations.

{0.913793, 0.934426, 0.939394, 0.896552, 0.936508, 0.955882, 0.921875, 0.875, 0.924528, 0.923077, 0.983871, 0.898551, 0.903226, 0.964286, 1, 0.95082, 0.870968, 0.940299, 0.919355, 0.931034, 0.963636, 0.919355, 0.916667, 0.962963, 0.916667, 0.890909, 0.980769, 0.935484, 0.984848, 0.958904, 0.916667, 0.9, 0.876923, 0.983333, 0.965517, 0.916667, 0.955224, 0.9375, 0.9375, 0.861538, 0.984375, 0.82, 0.936508, 0.885714, 0.915254, 0.901639, 0.857143, 0.936508, 0.935484, 0.919355}

Thus we may conclude that more than 90% of the "successful" simulations guaranteed the conclusion in Section 2.

4.2. Simulation when -10<k<0

In this subsection, we conduct a simulation to compare the desirability of taxes to provide the public goods by selecting parameters randomly when $k<0$. It was ascertained first that when k=-2 (a_1=1/8, b_1 =4/5, a_2=5/6, b_2 =1/7, a_2=2/7, b_3 =5/7, L_e=348, K_e=878, α_L=1/2, α_K=1/2, β_L=4/9, and β_K=5/9, k=-2, γ_y=13, γ_x=9, and γ_z=4), we have $Gini^L < Gini^I$ and $u_L*+u_K*+u_{E1}*+u_{E2}* > u_L*^I+u_K*^I+u_{E1}*^I+u_{E2}*^I$, with 22 equilibrium conditions satisfied. Thus, in this specified case, Lindahl tax is more desirable than income tax and commodity tax. This conclusion is completely opposite to the one in 4.1. In this specified case, the poll tax is feasible, but the comparison between the poll tax and other taxes was impossible. Note that $k<0$ does not violate the decreasing marginal rate of substitution.

In what follows, first, 100 tuples of parameters for $\{a_1, b_1, a_2, b_2, a_3, b_3, L_e, K_e, \alpha_L, \alpha_K, \beta_L, \beta_K, k, \gamma_y, \gamma_x, \gamma_z\}$ are selected randomly, where $a_i+b_i<1$, i =1, 2, $a_3+b_3=1$, $\alpha_L+\alpha_K=1$, $\beta_L+\beta_K=1$, and integer k with $-10<k<0$ are satisfied and a_i, b_i and α_L etc. are expressed by n/m for integers n and m which belongs to [1, 10], L_e and K_e are integers belonging to [1, 1000], and γ_y, γ_x, γ_z are integers belonging to [1, 10]. Among 100 simulations only 33 cases satisfied 22 equilibrium conditions. The reason for the smallness of the number stems from the Newton method itself, as explained in the previous subsection. We repeated this 100-simulation session 50 times. The following data shows the number of simulations in each session which satisfy 22 equilibrium conditions.

{28, 37, 34, 35, 32, 29, 26, 26, 27, 34, 28, 34, 30, 26, 28, 25, 32, 33, 27, 31, 25, 29, 27, 36, 28, 32, 23, 32, 25, 37, 33, 33, 28, 21, 29, 25, 23, 30, 31, 33, 34, 25, 32, 33, 34, 31, 30, 32, 28, 32}

Even though the probability of "successful" Newton method convergence is approximately 30%, among the "successful" simulations the probability of the occurrence of $Gini^L < Gini^I$ and $u_L^*+u_K^*+u_{E1}^*+u_{E2}^* > u_L^{*I}+ u_K^{*I}+ u_{E1}^{*I}+u_{E2}^{*I}$ is quite high.

For the above-mentioned repeated 50 sessions, the following data shows the shares of the cases whose conclusion is completely opposite to the one in 4.1 among the "successful" simulations with 22 equilibrium conditions satisfied.

{0.928571, 0.972973, 0.970588, 1, 0.9375, 0.931034, 1, 1, 1, 0.970588, 1, 1, 1, 0.961538, 0.964286, 0.96, 1, 0.969697, 1, 1, 1, 1, 0.962963, 0.944444, 0.964286, 0.96875, 0.913043, 0.9375, 1, 0.945946, 1, 1, 0.964286, 1, 1, 0.92, 1, 0.966667, 0.967742, 1, 0.970588, 1, 1, 1, 0.970588, 1, 0.966667, 0.96875, 0.928571, 0.96875}

Thus, more than 95% is the probability of the occurrence of $Gini^L < Gini^I$ and $u_L^*+u_K^*+u_{E1}^*+u_{E2}^* > u_L^{*I}+ u_K^{*I}+ u_{E1}^{*I}+u_{E2}^{*I}$. (The computation in Section 4 was conducted in Fukiharu[6].)

5. Walrasian tatonnement: part III

Specifying parameters on production and utility functions and initial endowments randomly, Fukiharu[1] showed that when $0<k<1$, the income tax (and proportional commodity tax) is more desirable than the Lindahl tax from the fairness and efficiency viewpoints with high possibility of non-existence for poll tax general equilibrium. However, when $k<0$, specifying parameters on production and utility functions and initial endowments randomly, Fukiharu[1] showed that the Lindahl tax is more desirable than the income tax (and proportional commodity tax) from the fairness and efficiency viewpoints with high possibility of existence for poll tax although the comparison between the poll tax and other taxes are impossible from the two viewpoints. Thus, in Fukiharu[1] the comparison completely depends on the substitution parameter of the CES utility function. In this chapter, stability analysis is first conducted on the Walras-Lindahl Mechanism. Samuelson[3] established a tradition in economics of deriving conclusions from the stability conditions on the markets. In other words, the economic conclusions must be compatible with stability of the markets.

5.1. Stability of Walras-Lindahl mechanism

As shown in Section 2, the solution to the system of general equilibrium: (2) ~ (6), can be derived by applying the Newton method, while it is also derived by the Walras-Lindahl mechanism, (7). It is ascertained that when $0<k<1$, (7) is globally stable, in the sense that starting from whatever the initial position of parameters, the dynamic process converges to the GE prices with optimal public good. This is not the case when $k<1$. In 4.2, when k=-2 (a_1=1/8, b_1 =4/5, a_2=5/6, b_2 =1/7, a_2=2/7, b_3 =5/7, L_e=348, K_e=878, α_L=1/2, α_K=1/2, β_L=4/9, and β_K=5/9, k=-2, γ_y=13, γ_x=9, and γ_z=4), we had $Gini^L<Gini^I$ and $u_L^*+u_K^*+u_{E1}^*+u_{E2}^*>u_L^{*I}+u_K^{*I}+u_{E1}^{*I}+u_{E2}^{*I}$. For this specification (7) is not globally stable, whereas it is locally stable. Indeed, on (7), starting from $\theta_1[0]$=5/10, $\theta_L[0]$=1/10, and $\theta_K[0]$=3/10, the trajectory of $\Theta[t]=\theta_1[t]+\theta_L[t]+\theta_K[t]$ reaches 1 when t=0.0010, and the dynamic process stops there. The actual trajectory of $\Theta[t]$ in this simulation is provided in Figure 1.

Naturally, if we select the initial values sufficiently close to the GE prices with public good, the dynamic process is stable. Thus, when $k<0$, the Walras-Lindahl mechanism is locally stable. This stability analysis may well reflect the high convergence result on the Newton method for $0<k<1$ and the low convergence result for $k<0$.

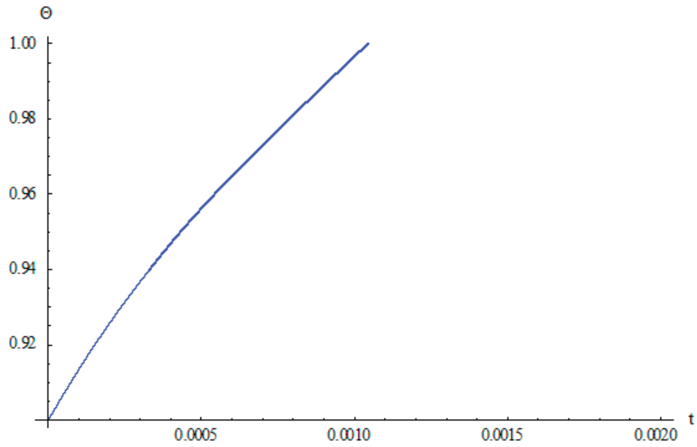

Fig.1: The trajectory of $\Theta[t]=\theta_1[t]+\theta_L[t]+\theta_K[t]$

5.2. Walrasian tatonnement process to compute the rate of income tax

In spite of the well-known usefulness in the *theoretical* application, the Newton method has a fatal flaw in the *actual policy* application. When the utility functions and production functions are known to the policy makers, they can easily compute the solution to (2) ~ (6) by simply applying the Newton method. In the actual world, however, it is quite difficult to estimate those functions, and the Walras-type tatonnement process is far more useful, in the sense that the policy makers' role is to announce the candidate for the solution to the economic agents such as consumers, and they just have to change it according to (7) when it is not the solution.

In the case of computing the rate of income tax in Subsection 3.2, the corresponding Walrasian tatonnement process may be the following system of differential equations.

$$\begin{aligned} &\mathrm{d}p_y[t]/\mathrm{d}t= y_L^{\mathrm{D}} + y_K^{\mathrm{D}} + y_{E1}^{\mathrm{D}} + y_{E2}^{\mathrm{D}} - y^{\mathrm{S}} \\ &\mathrm{d}p_x[t]/\mathrm{d}t= x_L^{\mathrm{D}} + x_K^{\mathrm{D}} + x_{E1}^{\mathrm{D}} + x_{E2}^{\mathrm{D}} - x^{\mathrm{S}} \\ &\mathrm{d}w_K[t]/\mathrm{d}t= K_1^{\mathrm{D}} + K_2^{\mathrm{D}} + K_3^{\mathrm{D}} - K_e \\ &\mathrm{d}\tau_I[t]/\mathrm{d}t= p_z z - \tau_I\,(w_L L_e + w_K K_e + \pi_1 + \pi_2) \end{aligned} \tag{16}$$

In (16), note that since the constant returns to scale assumption on the public good sector requires profit to be zero at the general equilibrium $p_z\,z= w_L L_3^{\mathrm{D}}+w_K K_3^{\mathrm{D}}$ must hold at GE and w_L=1.

It is possible to show in simulation that when 0<k<1, (16) is globally stable. (16) is locally unstable, however, when k<1. In the same parametric specification as in 4.2 and 5.1, the set of Eigen-values on the Jacobian matrix for (16) is {-24237.6, 3316.82, -652.992, -202.928}. As is well-known all the elements in the set must be negative in order for (16) to be locally stable.

As a conclusion, we may assert that, independent of k, the income tax (and proportional commodity tax) is more desirable than the Lindahl tax from the fairness and efficiency viewpoints with high possibility of non-existence for poll tax general equilibrium. (The computation in Section 5 was conducted in Fukiharu[7].)

6. Conclusions

It is a tradition in economics to take account of stability condition in examining the comparative statics. The aim of this paper is to resort to this tradition in examining the problem of whether the income taxation is superior to the Lindahl taxation or not. In Fukiharu[1], the present author derived the opposing conclusions on this problem depending on the parametric specification. In other words, when the substitution parameter on the CES utility function is positive the income taxation tends to be superior to the Lindahl taxation, while the opposite conclusion holds when the substitution parameter is negative. Fukiharu[1] computed the general equilibrium prices and tax rates in terms of either the Newton method or the Walras-Lindahl mechanism. In the present paper, on the one hand, those prices and taxes are computed solely in terms of Walras-Lindahl mechanism, and derived an interesting result, in which the Walras-Lindahl mechanism tends to be globally stable when it computes the Lindahl tax rates for providing the optimum public good as well as in computing the income tax rate in sustaining the optimum level of public good, so long as the substitution parameter is 1/2. The global stability is quite useful property in computing GE prices and tax rates, since whatever the initial positions concerning prices and tax rates may be, the trajectories on the differential equations converge to the GE prices and tax rates. On the other hand, when the substitution parameter is -2, the Walras-Lindahl mechanism tends to be locally stable when it computes the Lindahl tax rates for providing the optimum public good and it is unstable in computing the income tax rate in sustaining the optimum level of public good. The local stability is far less useful in computing the GE prices and tax rates, since it is required to find a suitable neighborhood of those GE prices and taxes, from which the convergent initial positions are selected. This problem is similar to the Newton method. In applying the Newton method it is a difficult task to find the initial points which guarantee the convergence to the equilibrium. Indeed, Fukiharu[1] adopted the two-stage search: *i.e.* he first computed the Lindahl taxes and prices in terms of Walras-Lindahl mechanism, proceeding to the Newton method in computing the income tax rate by selecting those GE prices as the initial points in the Newton method.

The result in this paper may support the income taxation in providing the optimum level of public good. Some may well oppose to this argument, since if we utilize the Newton method we could compute the GE prices and taxes. However, this opposition could be applied to the tradition of economics. Even if the dynamic system is unstable we might compute the new equilibrium so long as we know, say, demand and supply functions. The problem of the Newton method, however, lies in this point. In the Newton method, the planner must know the functional form of equations. Meanwhile, in Walras-Lindahl mechanism, the planner does not have to know the functional form of equations. What he must do is to announce the prices and tentative tax rates and modify them when the demand is not equal to supply for at least one item. The required information is very small compared with the Newton method in computing equilibrium.

Thus, we may conclude that the income taxation tends to be superior to the Lindahl taxation independent of the parametric specification. The GE model in the present paper is constructed under the framework of one public good and two private goods. The extension to the GE model with more than two private commodities will be attempted in the succeeding research.

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